

Reciprocal Trigonometric Identities

These notes are intended as a companion to section 7.3 (p. 605 – 611) in your workbook. You should also read the section for more complete explanations and additional examples.

Identities

An **identity** is a statement of equality that is true for all possible values of the variables involved. For example, the equation

$$x + 2 + 5 - 4 - x = 3$$

is true for all values of x . Thus, it is an identity.

A **trigonometric identity** is an identity that involves trigonometric functions. For example, the equation

$$\cos x = \sin\left(x + \frac{\pi}{2}\right)$$

is true for all values of x .

During the trigonometry unit we have already seen a handful of trigonometric identities.

Reciprocal Identities

The reciprocal identities result from the definitions of the reciprocal trigonometric functions:

$$\csc x = \frac{1}{\sin x} \quad (\sin x \neq 0) \qquad \sec x = \frac{1}{\cos x} \quad (\cos x \neq 0)$$

$$\cot x = \frac{1}{\tan x} \quad (\sin x \neq 0 \text{ and } \cos x \neq 0)$$

Quotient Identities

The quotient identities are a result of the definitions of tangent and cotangent, combined with our understanding of the unit circle.

$$\tan x = \frac{\sin x}{\cos x} \quad (\cos x \neq 0) \qquad \cot x = \frac{\cos x}{\sin x} \quad (\sin x \neq 0)$$

Verifying and Proving Identities

A trigonometric identity can be **verified** by substituting a value for the variable. A verification shows that the identity is true for a single value. It does not prove the identity is true for all values.

In order to **prove** an identity, it must be shown that one side is equal to the other side, or that both sides are equal to the same expression.

One of the most useful strategies for proving identities is to try writing all trigonometric functions in terms of either sine or cosine.

Example 1 (sidebar p. 607)

For each identity below:

- i) Verify the identity for $\theta = 30^\circ$.
- ii) Prove the identity.

a) $(\sec \theta)(1 + \cos \theta) = 1 + \sec \theta$

b) $1 - \tan \theta = \frac{\cot \theta - 1}{\cot \theta}$

Example 2 (sidebar p. 609)

For each identity below:

- i) Determine the non-permissible values of θ .
- ii) Prove the identity.

a)
$$\frac{\cot \theta}{\csc \theta} = \cos \theta$$

b)
$$\cos \theta = \frac{1 + \cos \theta}{1 + \sec \theta}$$

Example 3 (sidebar p. 610)

Use algebra to solve each equation over the domain $0 \leq x \leq 2\pi$.

a) $2 \sin x = 3 + 2 \csc x$

b) $\sin x = \cos x$

Homework: #3, 5, 6, 7ii, 8ii, 9ii, 10 in the exercises (p. 611 to 618). Answers on p. 619.